Home Production, Employment, and Monetary Policy

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Abstract

I extend the textbook Dynamic New Keynesian (DNK) model to include home production and labor supply decisions along both extensive and intensive margins. Home production introduces an asymmetrical effect of wage changes on the employment rate and average labor hours. As a result, the path of total labor hours over the business cycle becomes ambiguous. When the elasticity of substitution between home and market goods is above a threshold value of 2, the aggregate hours become procyclical. In contrast, the total labor hours are always countercyclical in the textbook model. This discrepancy is important for monetary policy: if a central bank excludes household production from its analysis, it mismeasures the output gap. The resulting welfare loss equals 0.013 percent in terms of consumption equivalent.

Keywords: Home production, extensive margin, monetary policy

1. Introduction

In this paper, I ask how home production – activities such as meal preparation, laundry, and lawn care – affects the decisions over the two margins of labor supply in the presence of nominal rigidities.[1] I find that when the opportunity cost of home production increases, individuals work more days per period (extensive margin) while choosing slightly shorter workdays (intensive margin). As a result, total hours of work increase in response to a market TFP shock, which is the opposite of what happens in the textbook DNK model. Moreover, adding the home sector or the two margin labor supply to the textbook model separately does not change the typical outcome of the textbook model. It turns out, home production has a stronger effect on employment than it does on hours. An increase in wages results in movement from home production to market work, which affects both participation and hours. Additionally, higher wages reduce the opportunity cost of each workday, which promotes further substitution from the home sector to the market sector – but only along the extensive margin. This behavior manifests in the "hump-shaped" response of total market hours to TFP changes. This

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[1]Home production does not include childcare because the income elasticity of time spent on child care is positive, while the income elasticity of time dedicated to home production is negative.

Table 1: Daily time allocation of employed individuals, % of daily hours

	market work	home production	leisure and sports	personal care and sleep
Workday	34.8	6.7	13.3	35.9
Day off	0	16.5	26.4	42.5

The table displays averages across 88,109 employed individuals ages 18-65 during 2003-2014. Market work includes time spent on commuting and other work-related activities. Home production includes related travel and wait time. Other includes time spent on meals and on miscellaneous activities.
*Source: own analysis of ATUS data.

channel is present only in a model that *jointly* considers two margins of labor supply and home production.

The discrepancies in the behavior of aggregate hours could lead to inconsistent monetary policy recommendations, depending on the choice of the theoretical model that is used to estimate the flexible level of output. The one-sector model and the home production model imply different estimates of the flexible output level. In particular, the flexible level of output in the home production model is more responsive to shocks. If the one-sector estimate is used, the projected output gap is overestimated, which leads to an over-adjustment of the nominal rate. As a result, the welfare loss in consumption equivalent terms is 0.013%.

Workhorse macroeconomic models treat employment and hours as perfect substitutes. However, a vast empirical literature suggests that this assumption is not reasonable. A recent study by Aguiar et al. (2013) shows that almost 75 percent of the decline in total work hours during the Great Recession is due to a drop in employment, and only 25 percent of the decline is due to a decrease in the per-worker hours. This result implies that the distinction between extensive and intensive margins of labor supply is important in the business cycle context. To relax the assumption about perfect substitutability between participation and hours of work, I introduce a fixed cost related to each day of work. An example of such cost is time spent commuting to/from a job. Many authors have extensively discussed the existence and importance of these costs, including Kydland and Prescott (1989) and Hansen (1985).

Another time allocation decision that is relevant to the business cycle is the time spent on home production. Since market goods are substitutes with consumption of the home-produced good, individuals can adjust their time input between the two sectors depending on the relative marginal cost of the two activities. Indeed, Aguiar et al. (2013) find that home production absorbs around 30 percent of foregone work hours at business cycle frequencies. I include the home sector in my model, picking an empirically estimated benchmark value of 2.5 for the elasticity of substitution between home and market goods. I conduct sensitivity analysis for this parameter and find that as long as it is above 1.2, employment is procyclical.

I analyze American Time Use Survey (ATUS) data to identify whether the allocation of time between home and market work is related to the extensive margin adjustment of labor supply. If the two decisions are independent, the daily time spent on home work should not differ significantly between workdays and off days. Table 1 shows that this is not the case for employed individuals. In fact, the share of time spent on home production during workdays is at 6.7 percent, whereas on days off it reaches 16.5 percent. Additionally, once individuals choose whether to work on a given day or not, their average workday length is 8.4 hours. On a day off, this time is split between home production, leisure, and personal care (includes sleep), such that the time spent on each activity increases.

The above data facts indicate that labor supply decisions are more complex than the ubiquitous in macroeconomic models labor-leisure choice. I write down my model (two-margin-home-production model) such that it can be reduced to three special cases: the textbook DNK model, the one-margin-home-production model, and the DNK model with two-margin labor supply and no home production (the "one-sector" model). I use log-utility of consumption such that the income and substitution effects of wage changes on the labor supply exactly offset each other. The reason hours are always countercyclical in the textbook DNK model is the presence of additional income effect that rises because the agent receives additional income – the profit from firm ownership. In a model with sticky prices this profit is large and, when transferred to households, motivates them to work less. Unless a feature that will offset this effect is introduced to the model

I compare the responses of variables to one standard deviation market TFP shock in my model to those in the one-sector model. The home sector becomes relatively less productive, which motivates agents to substitute away from home-produced goods towards the market-produced good, which implies a decrease in total time spent on home production and an increase in total market hours. This is the relative productivity effect which symmetrically affects both margins of labor supply. Additionally, a positive TFP shock makes the market participation less costly, which results in a switch between workday hours and employment. As a result, an asymmetrical response of extensive versus intensive margins of labor is introduced to the model. This asymmetry does not exist if there is no home production and no extensive margin in the model.

I discuss how a central bank should measure the output gap and the implications of this measure for welfare. This number is significant, considering the fact that this loss arises purely from the measurement error and not from a choice of a rule. Solving for the optimal coefficients on inflation and the output gap shows that when the output gap is mismeasured, the optimal coefficient on the gap is zero. However, if the output gap is measured correctly, the optimal coefficient is 1. This result implies that the gain from knowing the true output gap is substantial.

Benhabib et al. (1991) and Greenwood and Hercowitz (1991) were the first to consider home production within a real business cycle model. Their contribution has been complemented by work such as Perli (1998) and Campbell and Ludvigson (2001). Despite the fact that the real business cycle literature shows that home production is important, the implications of home production for monetary policy have been relatively unexplored. Only recently did Ngouana (2012) and Lester (2014) consider home sector within a New Keynesian model. The results from their simulations, however, are very different from the results in this paper because I include an additional margin of adjustment, which turns out to be critical for the labor supply decisions in a model with home production.

2. The Model

Time is discrete and has an infinite horizon. Each period consists of a large number of days. A representative agent engages in two types of production - market and home - and adjusts its labor supply along two margins, extensive (number of workdays) and intensive (hours per day). Labor-hiring firms do not differentiate between extensive and intensive margins of labor supply. The firm side of the economy is identical to that in the canonical DNK model. Home production is not paid and for self-consumption only. I refer to a day spent in market production as a workday and all other days as days off.

2.1. Representative agent

The representative agent has preferences over consumption and leisure and allocates his/her time between market work, home production, and leisure. The agent's total consumption is a CES aggregate of the market good consumption (g_t^M) and the home good consumption (g_t^H):

$$c_t = \left[\tau \left(g_t^M \right)^{\frac{\xi-1}{\xi}} + (1-\tau) \left(g_t^H \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \tag{1}$$

where τ is the importance of the market good to the agent and ξ is the elasticity of substitution between the two goods.

To highlight the importance of the time allocation between workdays and days off, I introduce differentiated leisure in the model. On a given day, the agent's utility is :

$$U\left(c_t, l_t^w, l_t^o\right) = \begin{cases} u\left(c_t\right) - v_1\left(1 - l_t^w\right) - \chi\left(e_t\right) & \text{if workday} \\ u\left(c_t\right) - v_2\left(1 - l_t^o\right) & \text{if day off} \end{cases} \tag{2}$$

where l_t^w is the average hours of leisure on workdays, l_t^o is the average hours of leisure on days off, $v_1(\cdot)$ and $v_2(\cdot)$ are increasing and twice continuously differentiable functions, e_t is the fraction of workdays in a period, and $\chi(e_t)$ is a fixed cost associated with each workday. An example of such cost is one's time spent commuting to/from a job.[2] The function $\chi(e_t)$ is twice continuously differentiable and increasing in e_t.

Motivated by the fact that employed individuals spend 2.5 times fewer hours on home production during workdays, I make a simplifying assumption that home production can occur only on days off. The daily time endowment is normalized to 1. The time constraint can then be written as:

$$1 = \begin{cases} h_t^M + l_t^w & \text{if workday} \\ h_t^H + l_t^o & \text{if day off} \end{cases} \tag{3}$$

where h_t^M and h_t^H are the average daily hours spent working in the market sector and the home sector respectively. Taking an average over all days in a period yields an expression for the average daily utility:

$$U\left(c_t, h_t; e_t\right) = u\left(c_t\right) - e_t \cdot v_1\left(h_t^M\right) - (1-e_t) \cdot v_2\left(h_t^H\right) - e_t \cdot \chi\left(e_t\right) \tag{4}$$

[2]The importance of such costs has been discussed by many authors, including Kydland and Prescott (1989), and Hansen (1985).

Every period, the agent solves the following problem:

$$\max_{\substack{c_t,\, g_t^M,\, g_t^H,\, b_{t+1} \\ e_t,\, h_t^M,\, h_t^H}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(c_t, e_t, h_t^M, h_t^H\right)$$

$$\text{s.t.} \quad g_t^M + b_{t+1} \leq \frac{1+i_t}{1+\pi_{t+1}} b_t + w_t e_t h_t^M + \Pi_t \tag{5}$$

$$g_t^H = (1-e_t) h_t^H \tag{6}$$

where b_{t+1} is the current period bond purchases, i_t is the nominal interest rate, π_{t+1} is the expected inflation rate, w_t is the real wage, and Π_t is the real profits received from firm ownership. Equation (5) is the budget constraint that states that the agent allocates the sum of wage income, interest income, and profits between market good expenditures and expenditures on new bond purchases. Equation (6) is the home production function. I assume the following functional forms throughout the paper:

$$\begin{aligned} u(c_t) &= \ln c_t \\ v_1\left(h_t^M\right) &= \tfrac{B}{1+\eta}\left(h_t^M\right)^{1+\eta} \\ v_2\left(h_t^H\right) &= \tfrac{A}{1+\epsilon}\left(h_t^H\right)^{1+\epsilon} \\ \chi(e_t) &= \tfrac{F}{1+\sigma} e_t^{\sigma} \end{aligned}$$

where η is the inverse Frisch elasticity of labor supply over the intensive margin, ϵ is the inverse elasticity of home to market hours, and σ, A, B, F are constants that will be pinned down by the steady state.

2.2. Market Production

The market side of the economy replicates the canonical DNK model. Production happens in two phases.

Intermediate goods are first manufactured by monopolistically competitive producers and then combined into the final good via the CES technology:

$$y_t = \left[\int_0^1 y_{j,t}^{\frac{\nu_t - 1}{\nu_t}} dj\right]^{\frac{\nu_t}{\nu_t - 1}} \tag{7}$$

where ν_t is the price markup shock. Intermediate goods are produced according to a linear production function:

$$y_{j,t} = z_t H_{j,t} \tag{8}$$

where $H_{j,t}$ is the labor hours employed by producer j, and z_t is the aggregate productivity shock.

Every period, a fraction $1-\theta$ of intermediate producers can choose their prices optimally, and a fraction θ has to keep the last period's price. The resulting aggregate price level is defined by:

$$p_t^{1-\nu_t} = (1-\theta)\left[p_t^{\#}\right]^{1-\nu_t} + \theta\left[p_{t-1}\right]^{1-\nu_t} \tag{9}$$

where $p_t^{\#}$ is the common reset price charged by the price-updating producers.

2.3. Central Bank

The benchmark monetary policy is set via the standard Taylor rule:

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t^{gap}\right) \tag{10}$$

where ϕ_π and ϕ_y are the monetary policy parameters, and $y_t^{gap} \equiv \left(y_t - y_t^f\right)/y_t^f$ is the deviation of output from its flexible price counterpart.

2.4. Exogenous Processes

The market productivity shock and the price markup shock are the only exogenous variables in the model. I assume that they follow AR(1) processes with non-stochastic means:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t} \tag{11}$$

$$\ln \nu_t = (1-\rho_\nu)\ln \nu_{ss} + \rho_\nu \ln \nu_{t-1} + \varepsilon_{\nu,t} \tag{12}$$

2.5. Aggregation and Equilibrium

In equilibrium, bond holdings must be zero and the market output must be consumed:

$$b_t = 0 \tag{13}$$

$$y_t = g_t^M \tag{14}$$

The aggregate demand for labor is the sum of labor hours demanded by each intermediate good producer:

$$H_t = \int_0^1 H_{j,t}\, dj \tag{15}$$

Market clearing requires that the aggregate labor demand equals the aggregate market labor supply:

$$H_t = e_t h_t^M \tag{16}$$

The aggregate market output is then:

$$y_t = \frac{z_t e_t h_t^M}{d_t} \tag{17}$$

where $d_t = \int_0^1 \left[\frac{p_{j,t}}{p_t}\right]^{\nu_t} dj$ is the measure of the deadweight loss due to price dispersion.

I loglinearize the first order conditions around the steady state. The full set of equilibrium conditions is stated in Appendix A.

3. Discussion of the Model

I express the employment and workday hours in terms of market good consumption, home good consumption, and wages. In loglinearized form, the labor supply curves are:

$$\frac{1}{\Phi_I}\tilde{h}_t^M = -\frac{k}{\xi}\tilde{g}_t^M - \frac{\xi - k}{\xi}\tilde{g}_t^H + \tilde{w}_t \tag{18}$$

$$\frac{1}{\Phi_E(\xi)}\tilde{e}_t = -\frac{\psi + (1-\psi)k}{\xi}\tilde{g}_t^M + \frac{\psi - (1-\psi)(\xi - k)}{\xi}\tilde{g}_t^H + \tilde{w}_t \tag{19}$$

where k and ψ are constants, and Φ_I and $\Phi_E(\xi)$ are the Frisch elasticities of labor supply over intensive and extensive margins respectively.[3] These elasticities are defined as follows:

$$\Phi_I = \frac{1}{\eta}, \text{ and } \Phi_E(\xi) = \frac{1}{\sigma}\left[\frac{\eta}{1+\eta} - \frac{\psi\epsilon}{1+\epsilon}\right]^{-1}$$

The above conditions indicate one can see that the representative agent's behavior is non-standard in three ways. First, the supply of labor to the market sector is a function of the home good consumption. Second, there are two labor supply curves. Third, the total Frisch elasticity, $\Phi_I + \Phi_E(\xi)$, is not a free parameter – it depends on the elasticity of substitution between the home and market goods.

In a model that incorporates home production agents can allocate time between three activities (market work, home work, and leisure), whereas the textbook model features only two available activities (market work and leisure). Because of the additional margin of adjustment in the home production model, market output and consumption are more sensitive to technological improvements in either sector. This happens because the relative productivity of home sector changes as market wages change.

The second and third features, however, entail two additional channels through which home production affects the market labor supply – "the employment channel" and "the Frisch channel". Mathematically, the employment channel is represented by the ambiguous in sign coefficient on the home good consumption in equation (19), which means that the income effect of changing wages can be offset by the effect of changes in home good consumption. The Frisch channel is present only in equation (19) – the elasticity of substitution between the home and market goods will define how responsive the employment rate is to changes in wages. As a result, the adjustment of employment and average hours will be asymmetrical. This property is the positive contribution of this paper to the literature.

To gain better intuition about how the three channels described above interact, I consider two special cases of the model: the case when home produced goods and market produced goods are perfect complements, and the case when they are perfect substitutes.

3.1. Perfect complements ($\xi = 0$)

The consumption aggregator becomes of a Leontief form, and the representative agent's optimal behavior is to acquire the home and market goods in a fixed proportion:

$$\tilde{g}_t^H = \tilde{g}_t^M = \tilde{c}_t$$

The labor supply curves (18)-(19) become:

$$\frac{1}{\Phi_I}\tilde{h}_t^M = -\tilde{g}_t^M + \tilde{w}_t \tag{20}$$

$$\frac{1}{\Phi_E'}\tilde{e}_t = -\tilde{g}_t^M + \tilde{w}_t \tag{21}$$

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[3] $k = 1 + \tau(\xi - 1)\left[\frac{g_{ss}^M}{c_{ss}}\right]^{\frac{\xi-1}{\xi}}$, $\psi = \frac{1-\tau}{\tau w_{ss}}\frac{e_{ss}}{1-e_{ss}}\left[\frac{g_{ss}^H}{g_{ss}^M}\right]^{\frac{\xi-1}{\xi}}$

where $\Phi'_E = (1+\eta)/\sigma\eta$ is the Frisch elasticity over the extensive margin of labor in the perfect complements case.

The effect of home production on hours and employment ceases, and the labor supply curves become similar to those in the one sector model – daily hours and employment always move in the same direction after a shock happens. If additionally, $\sigma = 0$, then the behavior of aggregate hours replicates the behavior of average daily hours, and the model becomes the textbook DNK model.

3.2. Perfect substitutes, $\xi = \infty$

The consumption aggregator becomes linear in consumption of both goods:

$$\tilde{c}_t = \alpha \tilde{g}_t^M + (1-\alpha)\,\tilde{g}_t^H$$

where $\alpha = \tau g_{ss}^M / c_{ss}$ is a constant. The labor supply curves (18)-(19) reduce to:

$$\frac{1}{\Phi_I}\tilde{h}_t^M = -\,\tilde{c}_t + \tilde{w}_t \qquad (22)$$

$$\frac{1}{\Phi_E(\xi)}\tilde{e}_t = -\,(1-\psi)\,\tilde{c}_t + \tilde{w}_t \qquad (23)$$

Consider the supply curve for employment. The constant ψ is increasing in the elasticity of substitution ξ, therefore when home and market goods are perfect substitutes, the coefficient on consumption in the equation (23) is the least negative, i.e. the income effect from wage changes is the smallest. This is how home production influences the employment rate through the employment channel.

$\Phi_E(\xi)$ is increasing in the elasticity of substitution between home and market goods, ξ. Thus, in the perfect substitutes case, the implied extensive Frisch elasticity is the largest, and the employment rate responds to changes in real wages by the most amount. As a result, the size of the substitution effect becomes bigger than the size of the income effect, and the employment rate can increase in response to a positive market TFP shock. Since deviations in aggregate hours equal to the sum of deviations in average daily hours and employment rate, it is also possible for the total hours to increase following a positive market TFP shock.

To summarize, if the home and market goods are perfect complements, the model reduces to the one-sector model. Additionally, if $\sigma = 0$, the model reduces to the textbook DNK model. If the goods are perfect substitutes, home production affects the aggregate market hours through three different channels – the relative productivity channel, the Frisch channel, and the employment channel. The combined effect is the increase in the relative size of the substitution effect arising due to changing real wages. For some threshold value of Φ_E, the response of aggregate hours becomes procyclical. If, however, $\Phi_E = 0$ then the addition of the home sector to the textbook DNK model does not change the behavior of labor supply qualitatively. Therefore, *only when the agent can adjust along both margins of labor supply in the presence of home sector*, the behavior of aggregate hours becomes dramatically different from that in the textbook model.

The cases discussed above show the representative agent's behavior at the extremes. If the goods are imperfect substitutes, the response of aggregate market hours to shocks will lie between the two extremes.

Table 2: Estimates of ξ

Authors	ξ
McGrattan, Rogerson, and Wright (1997)	1.75
Rupert, Rogerson, and Wright (1995)	1.80
Chang and Schorfheide (2003)	2.30
Aguiar, Hurst, and Karabarbounis (2013)	2.50
Benhabib, Rogerson, and Wright (1991)	5.00

4. Calibration

American Time Use Survey (ATUS) Data

I use ATUS data for 2003-2014 to compute the target values for e_{ss}, h^M_{ss}, and h^H_{ss}. I restrict the sample to employed individuals between the ages 18 and 65, further removing observations collected during government holidays. In total, there are 88,109 individuals in the sample. Following Aguiar et al. (2013), I use adjusted ATUS weights. I also use their definitions of market work and home production.

On average, employed individuals work 4.78 days a week, which implies a participation rate of 68 percent (4.78/7 days). A typical person works 8.4 hours per workday (including time for traveling to/from work and other work-related activities), which is 35 percent of the available time in a day. Working individuals, on average, spend 3.9 hours per day off engaged in home production, which is 16.25 percent of the available daily time. I set the steady state employment rate to the average participation rate, and the steady states of market hours and average home hours to the average time shares of home production and market production.

Standard/Fixed Parameters

I set β to match the annual real interest rate of four percent. The steady state inflation is 0. Market TFP shock has a standard deviation of $\sigma_z = 0.01$ and autocorrelation of $\rho_z = 0.95$. Nominal rate persistence is set to $\rho = 0.8$. Persistence of the markup process is equal to $\rho_\nu = 0.7$. Firm's elasticity of substitution is $\nu = 5$, which implies the steady state markup of 25 percent, and the frequency with which firms update prices is $\theta = 0.85$.

For the elasticity of home to market hours, I use the Aguiar et al. (2013) value of $\epsilon = 0.5$. For the relative weight of market consumption in the CES aggregator, I use a value of $\tau = 2/3$. The results are robust to changes in this parameter, as long as it is above one-half, which is a parameter restriction implied by the first order conditions.[4]

The empirical estimates of the elasticity of substitution between home-produced and market-produced goods, ξ, range from 1.75 to 5. For benchmark calibration I use Aguiar et al. (2013) estimate of $\xi = 2.5$. In addition, in Table 2 I report results for other values of ξ.

[4] Empirical estimates for the U.S. are well above 0.5. See Rogerson (2008), McGrattan et al. (1997), and Moslehi et al. (2015).

Table 3: Benchmark Model Parameter Values

Parameter	Value	Interpretation
β	0.99	discount factor
e_{ss}	0.68	steady state employment rate
h_{ss}^M	0.35	steady state market hours
h_{ss}^H	0.1625	steady state home hours
τ	0.67	relative weight of market consumption
ξ	2.50	elasticity of substitution between market and home goods
ϵ	2.00	inverse of elasticity of home to market hours
Φ_I	0.22	Frisch elasticity over intensive margin
Φ_E	2.68	Frisch elasticity over extensive margin in HP model
ϕ_π	1.50	Taylor Rule weight on inflation
ϕ_y	0.50	Taylor Rule weight on output gap
ν_{ss}	5.00	elasticity of substitution between intermediate goods
θ	0.85	Calvo parameter
ρ_z	0.95	persistence of the productivity shock
σ_z	0.01	standard deviation of the productivity shock

The above parameters are used for the simulations in Section 5.

For the Frisch elasticities over intensive and extensive margins, Φ_I and Φ_E, I use estimated by Peterman (2016) values of 0.22 and 2.68 respectively, such that the total Frisch elasticity is equal 2.9.[5] Given Φ_I and Φ_E, I find the values for σ implied by (3):

$$\sigma = \frac{1}{\Phi_E}\left[\frac{\eta}{1+\eta} - \frac{\epsilon}{1+\epsilon}\frac{1-\tau}{\tau w_{ss}}\frac{e_{ss}}{1-e_{ss}}\left[\frac{g_{ss}^H}{g_{ss}^M}\right]^{\frac{\xi-1}{\xi}}\right]^{-1} = 0.84 \tag{24}$$

This pins down the value for the extensive Frisch elasticity in the one-sector model, $\Phi_E' = (\Phi_I + 1)/\sigma = 1.46$. Equation (24) implies that, for a given σ, the value of ξ is restricted such that the term in parentheses on the right-hand side is positive. The smallest value that implies a positive value of the extensive margin Frisch elasticity in the home production model is $\xi = 1.2$.

The preference parameters A, B, and F are pinned down by the parameters discussed above. Table 3 summarizes the full list of parameters.

5. Quantitative Results

Figure 1 depicts how variables in two different economies – one that has a home sector and one that does not – react to a positive market TFP shock. First, the response of employment to the TFP shock in HP model is of opposite sign than that in the 1S model. Second, the workday hours decrease in both models; however, the average hours in the HP model are almost three times as responsive as workday hours in the 1S model.

[5]Table 3 in Peterman (2016). The author uses a pseudo panel to estimate which includes hours fluctuations on both the intensive and extensive margins and broadens the scope of the sample to include all individuals between the ages of 20 and 65.

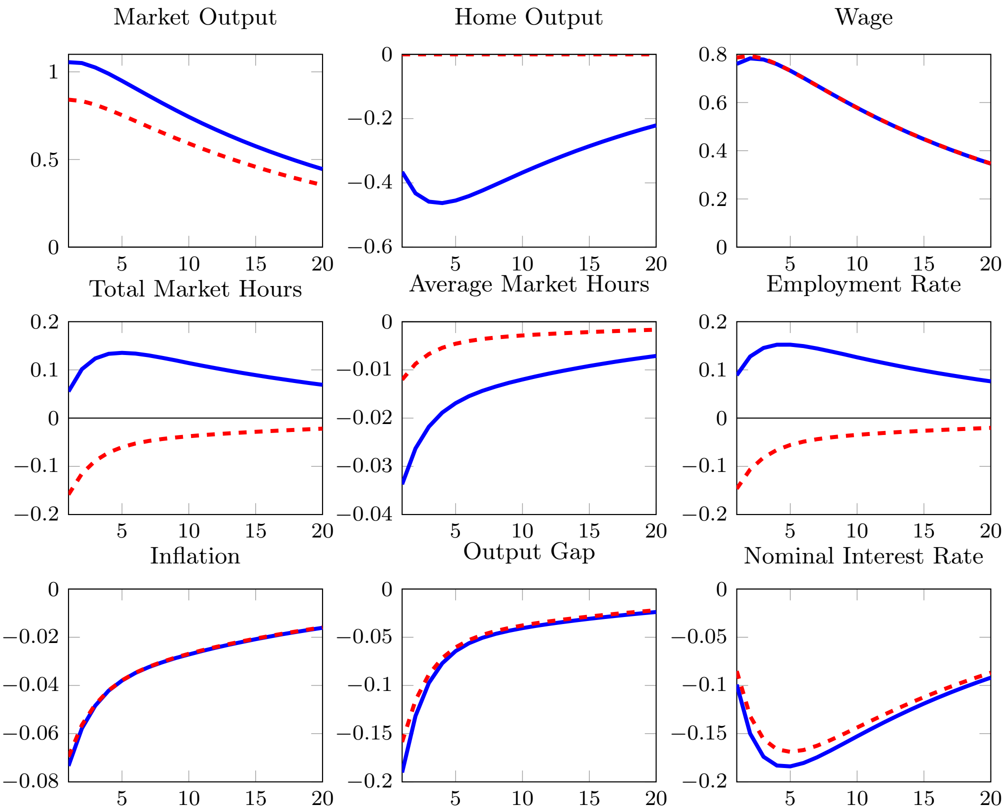

Figure 1: Response to a Market TFP Shock. The figure displays impulse responses to a one standard deviation positive TFP shock in the market sector. Solid blue line represents the model with home production and dashed red line represents the one-sector model. The units of horizontal axes are quarters. The units of vertical axes are percent deviations from steady state. Inflation and interest rate are annualized.

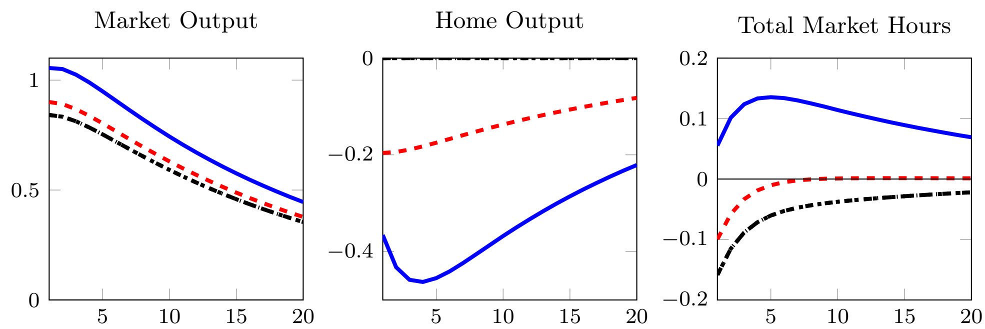

Figure 2: Response to a TFP Shock: Special Cases of the Model. The figure displays impulse responses to a one standard deviation positive TFP shock in the market sector. Solid blue line represents the two-margin-home-production model, dashed red line is Lester (2014) model, dotted black line is the one-sector model, dot-dash black line is the textbook model.

Table 4: Correlation of Labor Supply Variables with GDP Growth

Parameter	Data	No HP		HP	
		1M	2M	1M	2M
Total market hours	0.88	-0.92	-0.91	-0.46	0.98
Average workday hours	-0.08	n/a	-0.91	n/a	-0.97
Employment	0.83	n/a	-0.91	n/a	0.99

Third, aggregate hours and employment in the HP model are less responsive to the shock on impact, however, they increase in the consequent periods reaching their maximum in the fourth quarter. Finally, market output/consumption in the HP model rises by more than the shock size, whereas the output in the one-sector model is not as responsive to the TFP shock.

Figure 2 depicts the behavior of market output, home output, and the total market hours for the special cases of the model. Setting the Frisch elasticity over the extensive margin to zero reduces my model to Lester (2014). The aggregate market hours are countercyclical in this case. Setting the elasticity of substitution between the home and market goods to zero further reduces the model to the textbook DNK model. Only when I account for both margins of labor supply *and* home production, the behavior of aggregate variables becomes entirely different. Specifically, total hours become positively correlated with output. Table 4 lists the correlation coefficients based on the data and implied by the four models. Only the two-margin-home-production model can generate procyclical behavior of aggregate market hours because it introduces an asymmetrical effect of home production on market hours and employment, which is absent in the other three models.

Figure 3 shows the impulse responses for different values of the elasticity of substitution between market and home goods, ξ. When the two goods are perfect substitutes, aggregate hours rise by approximately 0.25 percent in response to a 1 percent increase in the market TFP, and a significant drop in home hours is observed. As the elasticity of substitution becomes smaller, market labor supply responds positively to an improvement in technology. Once the elasticity of substitution between the two goods is lower than 2, employment and aggregate hours become countercyclical. When the goods are perfect complements, the model approaches the one-sector model.

Figure 4 depicts the impulse responses for different values of the extensive margin Frisch elasticity. The low value of Φ_E imply the standard DNK behavior of variables. As the elasticity increases, the movements in workday hours are less predictive of changes in total hours. At the Frisch elasticity of 1, the aggregate hours become procyclical.

6. Policy Implications

To show that omission of home production in the monetary policy analysis has non-trivial consequences for the economy, I consider the following thought experiment. Recall that the output gap is the deviation of actual market output from the non-observable flexible level of output. To estimate the flexible output, a central bank needs to form a belief about which theoretical model represents the actual economy best.

If the central bank believes that the one-sector model represents the economy best, then the estimated flexible output is $y_t^{f,o}$. If the central bank believes that the model with home production is a better choice, then the estimated flexible output is $y_t^{f,hp}$. Regardless of which model is chosen, if it is, in fact, a true representation of the economy, the output gap is measured correctly, and the interest rate adjusts by the proper amount:

$$i_t^{hp} = \rho i_{t-1}^{hp} + (1-\rho)\left[\phi_\pi \pi_t^{hp} + \phi_y \left(\tilde{y}_t^{hp} - \tilde{y}_t^{f,hp}\right)\right] \tag{25}$$

or

$$i_t^{o} = \rho i_{t-1}^{o} + (1-\rho)\left[\phi_\pi \pi_t^{o} + \phi_y \left(\tilde{y}_t^{o} - \tilde{y}_t^{f,o}\right)\right] \tag{26}$$

However, when there is a mismatch between the central bank's belief and the actual economy, a problem arises. For example, if the home production model is reality, but the central bank chooses to estimate the flexible output based on the one-sector model, then the nominal rate will adjust according to:

$$i_t^{error} = \rho i_{t-1}^{hp} + (1-\rho)\left[\phi_\pi \pi_t^{hp} + \phi_y \left(\tilde{y}_t^{hp} - \tilde{y}_t^{f,o}\right)\right] \tag{27}$$

Depending on whether the output gap is overestimated or underestimated, the interest rate will adjust by too much or too little. As a result, monetary policy will not be as effective as it could be under the right model assumption.

To quantify this inefficiency, I derive a second order approximation of the household's objective function and equilibrium conditions. The expected lifetime utility of the household can be expressed in the recursive form:

$$v_{0,t} = U_t + \beta E_t v_{t+1}$$

where

$$U_t = \ln c_t - (1-e_t)\frac{A\left(h_t^H\right)^{1+\epsilon}}{1+\epsilon} - e_t\frac{B\left(h_t^M\right)^{1+\eta}}{1+\eta} - \frac{Fe_t^{1+\sigma}}{1+\sigma}$$

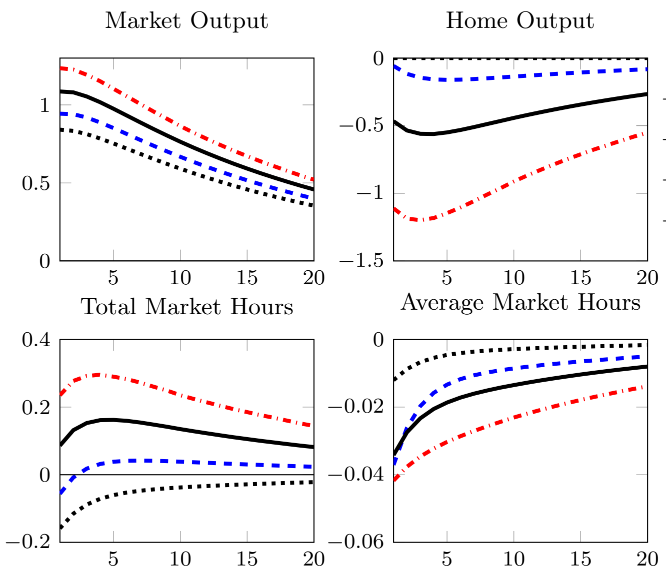

Figure 3: Sensitivity to Elasticity of Substitution, ξ. The figure displays impulse responses of variables to a one standard deviation positive TFP shock. Black dotted line represents IRFs for $\xi = 0$, blue dash line for $\xi = 1.5$, solid black line for $\xi = 2$, and red dash-dot line shows IRFs for $\xi = \infty$. The units of the vertical axes are percent deviations from the steady state, and the units of the horizontal axes are quarters.

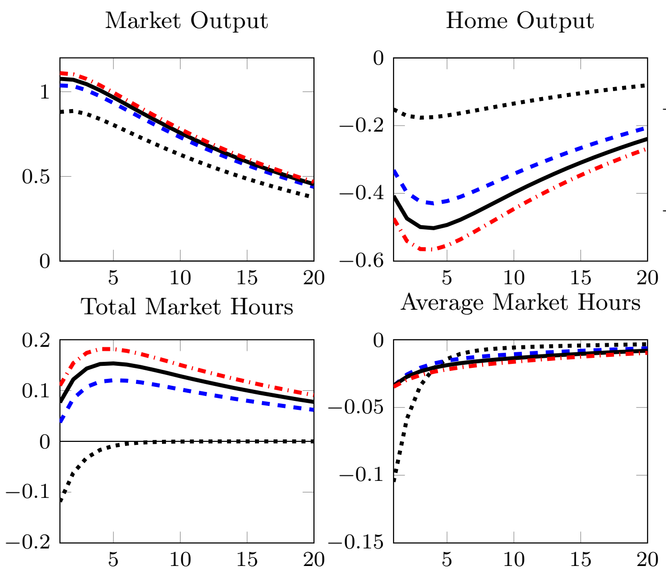

Figure 4: Sensitivity to Extensive Frisch Elasticity, Φ_E. The figure displays impulse responses of variables to a one standard deviation positive TFP shock. Black dotted line represents IRFs for $\Phi_E = 0.05$, blue dash line for $\Phi_E = 1$, solid black line for $\Phi_E = 4$, and red dash-dot line shows IRFs for $\Phi_E = 10$. The units of the vertical axes are percent deviations from the steady state, and the units of the horizontal axes are quarters.

Table 5: Consumption Equivalent, %

	$\sigma_\nu : 0\%$		$\sigma_\nu : 5\%$		$\sigma_\nu : 20\%$	
	correct	incorrect	correct	incorrect	correct	incorrect
	(1)	(2)	(3)	(4)	(5)	(6)
$\phi_y = 0.0$	0.030	0.030	0.314	0.314	1.386	1.386
$\phi_y = 0.1$	0.017	0.025	0.289	0.297	1.316	1.324
$\phi_y = 0.5$	0.003	0.016	0.312	0.325	1.481	1.493
$\phi_g = 0.1$	0.030	0.030	0.312	0.312	1.376	1.376
$\phi_g = 0.5$	0.030	0.030	0.304	0.304	1.337	1.337

Flexible price welfare level is $v^f_{0,t} = -215.04$
Taylor rule weight on inflation is kept constant at $\phi_\pi = 1.5$
(1)-(2): correspond to the case when output variability is 100% due to TFP
(3)-(4): 5% due to markup volatility and 95% due to TFP changes
(5)-(6): 20% due to the markup shock, and 80% due to TFP changes

Conditional on starting in the steady state, the second-order approximation of the value function is:

$$v_{0,t} \approx \bar{v} + \frac{1}{2}\Delta^2, \quad \bar{v} = \frac{U_{ss}}{1-\beta}$$

where Δ^2 is a vector of constant correction terms corresponding to the deviations of the expected value from its steady state. For easier interpretation of the welfare loss, I define consumption equivalent as a fraction Ω by which household's total consumption has to increase under a general interest rate regime to make the household indifferent between the general interest rate regime and optimal regime that results in flexible prices. Ω solves equation:

$$\sum_{t=0}^{\infty} \beta^t U\left((1+\Omega)\, c_t, e_t, h_t^H, h_t^M\right) = v_{0,t}^{opt}$$

where $v_{0,t}^{opt}$ is welfare level in the flexible-price world. One can solve for Ω:

$$\Omega = \exp\left[(1-\beta)\left(v_{0,t}^{opt} - v_{0,t}\right)\right] - 1 \tag{28}$$

Consider a market TFP shock. Table 5 provides results for different Taylor rule coefficients for the correctly and incorrectly specified output gap in the economy with the home production sector under the policy rules characterized by equations (25) and (27) respectively. Consider the first two columns.

If the gap is accurately specified, a conventional Taylor rule implies that consumption has to be increased by 0.003%. In contrast, the same rule with the misspecified gap results in the consumption equivalent of 0.016%. This difference in consumption equivalents arises purely from the measurement error and not from a choice of a rule (among the rules with non-zero response to the output gap). Notwithstanding the welfare loss caused by the measurement error, it is still better to respond to the mismeasured gap than not to respond to the mismeasured gap.

In addition to the technology shock, I consider a markup shock. Figure 5 and Table 5 display the results for the case when 5% of market output volatility is caused by the fluctuations in the price markup, and for the case when 20% of market output volatility

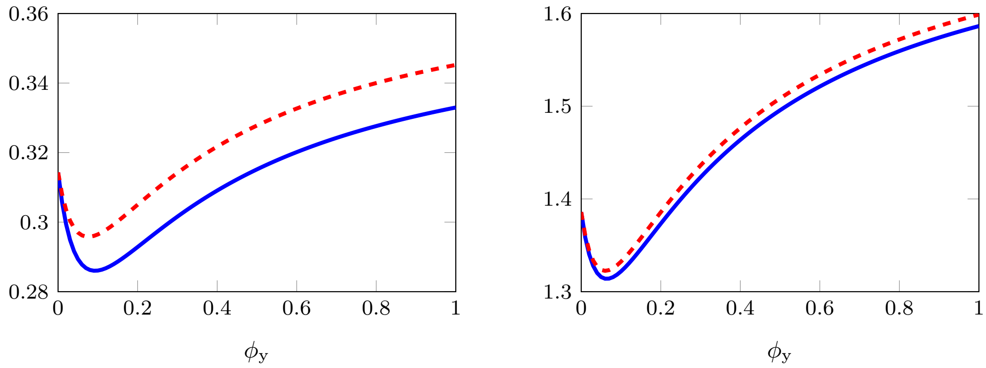

Figure 5: Relative Importance of Markup Shocks for Welfare. The figure depicts the welfare changes from responding to output gap about not responding, when the markup shock contributes to 5 percent of the output variability. The solid line represents the welfare changes from responding to the correct output gap, and the dashed line shows the welfare changes from responding to the incorrect output gap. Vertical axis is expressed in percentages, and the horizontal axis represents different weights on the output gap in the Taylor Rule.

is due to the volatility of the markup.[6] When the markup shock is present, welfare is no longer a monotonic function of the output gap coefficient. Specifically, responding to the output gap, both correctly and incorrectly measured, is welfare improving up to a certain point, but becomes welfare reducing beyond it. However, the introduction of markup shocks does not exaggerate the difference in the marginal welfare loss from the measurement error, i.e. the percentage point difference between the two lines on each panel of Figure 5 is the same for the three cases.

To see whether the central bank can avoid a welfare loss from the output gap misspecification, I consider a policy rule with a response to the output growth rather than the output gap. The results of this experiment are recorded in rows 4 and 5 of Table 5 and in Figure 6. In the absence of markup shocks, it is always welfare improving to respond to the incorrect gap than it is to respond to the output growth. However, when there is a markup shock, it is better to respond to the mismeasured gap only at lower coefficient values.[7]

Finally, I solve for the optimal coefficients on inflation and the output gap conditional on correctly perceiving and misperceiving the output gap.[8] The results are presented in Table 6. When the output gap is mismeasured and there is no markup shock, the optimal coefficient on the mismeasured gap is zero. However, if the output gap is measured correctly, the optimal coefficient is 1, which implies that the gain from knowing the true output gap is substantial. When a markup shock is introduced, the optimal coefficient on the mismeasured gap is no longer zero, and the optimal coefficient on inflation decreases.

[6] The the standard deviation of markup shock that results in 5% of the variability is 0.1417; the same for the 20% of the variability is 0.3088.

[7] A response to deviations of output from its steady state level had been considered as well – responding to the output growth is better.

[8] I set an upper bound on ϕ_π to 10.

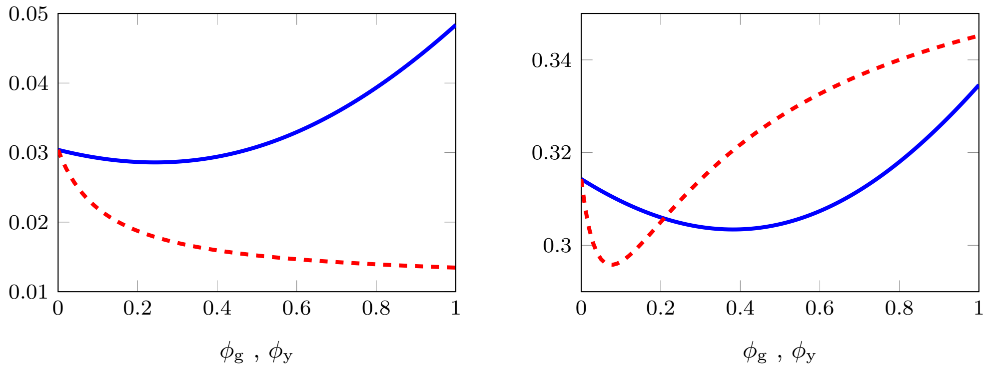

Figure 6: Responding to Output Gap versus Output Growth. The left panel depicts consumption equivalents when there is no markup shock, and the right panel is for the case when markup shock contributes to 5% of output variability. Blue lines represent response to growth, and dashed red lines show responses to the incorrect gap.

Table 6: Optimal policy rules

	σ_ν : 0%		σ_ν : 5%		σ_ν : 20%	
	ϕ_π	ϕ_y	ϕ_π	ϕ_y	ϕ_π	ϕ_y
correct	10	1.0	5.8	0.3	5.8	0.3
incorrect	10	0.0	6.0	0.3	5.8	0.3

Once the markup shock becomes larger, the optimal rule looks the same for both correct and incorrect gaps.

7. Conclusion

I relax the assumption that the participation rate and workday hours are perfect substitutes and show the presence of an important link between the employment and time spent in home production. When relative home sector productivity decreases, individuals switch from home production to market work. Simultaneously, an increase in wages reduces the opportunity cost of participation, which results in the substitution of daily work hours for more workdays per period. The aggregate hours increase following a productivity shock. In contrast, if either extensive margin of labor supply or home production are excluded from the model, aggregate hours always respond negatively to positive market TFP shocks, which is a general result in the textbook DNK model and existing DNK models with home production. Another noteworthy result indicated in this paper is that when the central bank mistakenly chooses a model without the home sector as a true representation of the economy, it estimates the output gap incorrectly. As a result, both market and home sectors contract, leading to nontrivial welfare losses.

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References

Aguiar, M., Hurst, E., Karabarbounis, L., Aug 2013. Time use during the great recession. American Economic Review 103 (5), 1664–1696.
URL http://dx.doi.org/10.1257/aer.103.5.1664

Benhabib, J., Rogerson, R., Wright, R., 1991. Homework in macroeconomics: household production and aggregate fluctuations. Staff Report, Federal Reserve Bank of Minneapolis 135, Federal Reserve Bank of Minneapolis.
URL http://ideas.repec.org/p/fip/fedmsr/135.html

Campbell, J., Ludvigson, S., 2001. Elasticities of substitution in real business cycle models with home production. Scholarly Articles 3163262, Harvard University Department of Economics.
URL http://ideas.repec.org/p/hrv/faseco/3163262.html

Chang, Y., Schorfheide, F., 2003. Labor shifts and economic fluctuations. Tech. rep.

Greenwood, J., Hercowitz, Z., December 1991. The Allocation of Capital and Time over the Business Cycle. Journal of Political Economy 99 (6), 1188–214.
URL https://ideas.repec.org/a/ucp/jpolec/v99y1991i6p1188-214.html

Hansen, G. D., November 1985. Indivisible labor and the business cycle. Journal of Monetary Economics 16 (3), 309–327.
URL http://ideas.repec.org/a/eee/moneco/v16y1985i3p309-327.html

Kydland, F. E., Prescott, E. C., 1989. Hours and employment variation in business cycle theory. Tech. rep.

Lester, R., 2014. Home production and sticky price models: Implications for monetary policy. Journal of Macroeconomics 41 (C), 107–121.
URL https://ideas.repec.org/a/eee/jmacro/v41y2014icp107-121.html

McGrattan, E. R., Rogerson, R., Wright, R., May 1997. An equilibrium model of the business cycle with household production and fiscal policy. International Economic Review 38 (2), 267–90.
URL http://ideas.repec.org/a/ier/iecrev/v38y1997i2p267-90.html

Moslehi, S., Tanaka, S., Moro, A., 2015. Does home production drive structural transformation? 2015 Meeting Papers 550, Society for Economic Dynamics.
URL http://ideas.repec.org/p/red/sed015/550.html

Ngouana, C. L., Aug. 2012. Household production, services and monetary policy. IMF Working Papers 12/206, International Monetary Fund.
URL http://ideas.repec.org/p/imf/imfwpa/12-206.html

Perli, R., February 1998. Indeterminacy, home production, and the business cycle: A calibrated analysis. Journal of Monetary Economics, Elsevier 41 (1), 105–125.
URL http://ideas.repec.org/a/eee/moneco/v41y1998i1p105-125.html

Peterman, W. B., 01 2016. Reconciling micro and macro estimates of the frisch labor supply elasticity. Economic Inquiry 54 (1), 100–120.
URL http://ideas.repec.org/a/bla/ecinqu/v54y2016i1p100-120.html

Rogerson, R., Apr 2008. Structural transformation and the deterioration of European labor market outcomes. J POLIT ECON 116 (2), 235?ĂŞ259.
URL http://dx.doi.org/10.1086/588029

Rupert, P., Rogerson, R., Wright, R., June 1995. Estimating substitution elasticities in household production models. Economic Theory 6 (1), 179–93.
URL http://ideas.repec.org/a/spr/joecth/v6y1995i1p179-93.html

Appendix A. Equilibrium Conditions

Loglinearized equilibrium conditions:

$$\eta\tilde{h}_t^M = \frac{1-\xi}{\xi}\tilde{c}_t - \frac{1}{\xi}\tilde{g}_t^M + \tilde{w}_t$$

$$\sigma\left[\frac{\eta}{1+\eta}\psi - \frac{\epsilon}{1+\epsilon}\right]\tilde{e}_t = \eta\psi\tilde{h}_t^M - \epsilon\tilde{h}_t^H$$

$$\epsilon\tilde{h}_t^H = (1-\xi)\,\tilde{c}_t - \tilde{g}_t^H$$

$$\tilde{c}_t = \alpha\tilde{g}_t^M + (1-\alpha)\,\tilde{g}_t^H$$

$$\tilde{g}_t^H = \tilde{h}_t^H - \frac{e_{ss}}{1-e_{ss}}\tilde{e}_t$$

$$\tilde{\lambda}_t = -\frac{1}{\xi}\tilde{g}_t^M - \frac{\xi-1}{\xi}\tilde{c}_t$$

$$\tilde{\lambda}_t = \mathbf{E_t}\tilde{\lambda}_{t+1} + i_t - \mathbf{E_t}\pi_{t+1}$$

$$\tilde{g}_t^M = \tilde{y}_t$$

$$\tilde{y}_t = \tilde{z}_t + \tilde{e}_t + \tilde{h}_t^M$$

$$\tilde{w}_t = \tilde{z}_t + \tilde{mc}$$

$$\pi_t = \frac{(1-\theta)\,(1-\theta\beta)}{\theta}\tilde{mc}_t + \beta\mathbf{E_t}\pi_{t+1} - \tilde{\nu}_t$$

$$y_t^{gap} = \tilde{y}_t - \tilde{y}_t{}^f$$

$$\tilde{z}_t = \rho_z\tilde{z}_{t-1} + \varepsilon_{z,t}$$

$$\tilde{\nu}_t = \rho_\nu\tilde{\nu}_{t-1} + \varepsilon_{\nu,t}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi\pi_t + \phi_y y_t^{gap}\right)$$

where:

$$\alpha = \tau\left[\frac{g_{ss}^M}{c_{ss}}\right]^{\frac{\xi-1}{\xi}}$$

$$\psi = \frac{\alpha}{1-\alpha}\frac{1-e_{ss}}{e_{ss}}w_{ss}$$

Nonlinear equilibrium conditions:

$$\tau \left[\frac{g_t^M}{c_t}\right]^{\frac{\xi-1}{\xi}} = \lambda_t g_t^M$$

$$(1-\tau)\left[\frac{g_t^H}{c_t}\right]^{\frac{\xi-1}{\xi}} = A(1-e_t)\left[h_t^H\right]^{\epsilon+1}$$

$$B\left[h_t^M\right]^{\eta} = \lambda_t w_t$$

$$\frac{\eta}{1+\eta}B\left[h_t^M\right]^{\eta+1} - \frac{\epsilon}{1+\epsilon}A\left[h_t^H\right]^{\epsilon+1} = Fe_t^{\sigma}$$

$$\lambda_t = \beta\frac{1+i_t}{1+\pi_{t+1}}\lambda_{t+1}$$

$$g_t^H = (1-e_t)h_t^H$$

$$c_t = \left[\tau\left(g_t^M\right)^{\frac{\xi-1}{\xi}} + (1-\tau)\left(g_t^H\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

$$g_t^M = y_t$$

$$y_t = \frac{z_t e_t h_t^M}{d_t}$$

$$d_t = (1+\pi_t)^{\nu_t}\left[(1-\theta)(1+\pi_t^{\#})^{-\nu_t} + \theta d_{t-1}\right]$$

$$w_t = \frac{z_t mc_t}{d_t}$$

$$1+\pi_t^{\#} = \frac{\nu_t}{\nu_t-1}\frac{x_t^1}{x_t^2}(1+\pi_t)$$

$$x_t^1 = \frac{mc_t y_t}{c_t} + \theta\beta E_t\left(\frac{1+\pi_{t+1}}{1+\pi_t}\right)^{\nu_t} x_{t+1}^1$$

$$x_t^2 = \frac{y_t}{c_t} + \theta\beta E_t\left(\frac{1+\pi_{t+1}}{1+\pi_t}\right)^{\nu_t-1} x_{t+1}^2$$

$$1+\pi_t = \left[(1-\theta)\left(1+\pi_t^{\#}\right)^{1-\nu_t} + \theta\right]^{\frac{1}{1-\nu_t}}$$

$$y_t^{gap} = \frac{y_t - y_t^f}{y_t^f}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t^{gap}\right)$$

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}$$

$$\ln \nu_t = (1-\rho_\nu)\ln \nu_{ss} + \rho_\nu \ln \nu_{t-1} + \varepsilon_{\nu,t}$$